The first result is the same as the average distance of a point in a circle from a point in its circumference. The second result is the same as the average distance between two points on the circumference of a circle.

Also solved by *F. P. MATZ*. Professor Matz in his solution did not go through the details of integration as Professor Zerr has done.

II. Solution by L. C. WALKER, A. M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Let P, Q be the random points, O the center of the hemisphere, and A the vertex. Put $OP=r$, $\angle POQ=\theta$, $\angle AOP=\varphi$, M_1=average length of the straight line, and M_2=average length of the arc of a circle, which joins the points.

Now while θ is constant, and $<\frac{1}{2}\pi$, and $\phi<\frac{1}{2}\pi-\theta$, for each position of P, Q may be taken anywhere in the circumference of a small circle whose pole is P, and radius $r\sin\theta$.

But when $\theta<\frac{1}{2}\pi$, and $\varphi>\frac{1}{2}\pi-\theta$, or when $\theta>\frac{1}{2}\pi$, and $\phi>\theta-\frac{1}{2}\pi$, for each position of P, Q may be taken anywhere in the arc of a small circle whose pole is P, and length $2r\sin\theta[\pi-\cos^{-1}(\cot\theta\cot\phi)]$.

Hence, if θ is of given value, and $<\frac{1}{2}\pi$, the number of ways the two points can be taken is

$$\int_0^{\frac{1}{2}\pi-\theta} 2\pi r\sin\theta.2\pi r\sin\phi.rd\phi+\int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi} 2r\sin\theta[\pi-\cos^{-1}(\cot\theta\cot\phi)].2\pi r\sin\phi.rd\phi$$

$$=4\pi r^3(\pi-\theta)\sin\theta.$$

If $\theta>\frac{1}{2}\pi$, the number of ways the two points can be taken is

$$\int_{\theta-\frac{1}{2}\pi}^{\frac{1}{2}\pi} 2r\sin\theta[\pi-\cos^{-1}(\cot\theta\cot\phi)].2\pi r\sin\phi.rd\phi=4\pi r^3(\pi-\theta)\sin\theta.$$

Hence, since the whole number of ways the two points can be taken is $4\pi^2r^4$, we have

$$M_1=\frac{1}{4\pi^2r^4}\int_0^{\pi} 4\pi r^3(\pi-\theta)\sin\theta.2r\sin\tfrac{1}{2}\theta.rd\theta=\frac{32r}{9\pi}, \text{ and}$$

$$M_2=\frac{1}{4\pi^2r^4}\int_0^{\pi} 4\pi r^3(\pi-\theta)\sin\theta.r\theta.rd\theta=\frac{4r}{\pi}.$$

MISCELLANEOUS.

101. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

A wire is laid along the surface of a right cone semi-vertical angle β so that it cuts the generators everywhere at a constant angle θ. Find the radius of curvature and radius of torsion.

Solution by the PROPOSER.

Let s=length of wire from origin to any point, ρ=radius of curvature, $1/\sigma$=radius of torsion. From problem 85, Calculus, No. 4, Vol. VI., we get $x=r\cos\varphi$, $y=r\sin\varphi$, $z=s\cos\beta\cos\theta$, $\varphi=\frac{\tan\theta}{\sin\beta}\log s$, $r=s\sin\beta\cos\theta$, $dx=\cos\varphi dr-r\sin\varphi d\varphi$,

$dy=\sin\varphi dr+r\cos\varphi d\varphi$, $dz=\cos\beta\cos\theta ds$, $d\varphi=\frac{\tan\theta}{\sin\beta}.\frac{ds}{s}$, $dr=\sin\beta\cos\theta ds$.

$\therefore$ $dx/ds=\sin\beta\cos\theta\cos\varphi-\sin\theta\sin\varphi$, $dy/ds=\sin\beta\cos\theta\sin\varphi+\sin\theta\cos\varphi$,

$dz/ds=\cos\beta\cos\theta$, $d^2x/ds^2=-\frac{\sin\theta}{s}\left(\sin\varphi+\frac{\tan\theta\cos\varphi}{\sin\beta}\right)$,

$d^2y/ds^2=\frac{\sin\theta}{s}\left(\cos\varphi-\frac{\tan\theta\sin\varphi}{\sin\beta}\right)$, $d^2z/ds^2=d^3z/ds^3=0$,

$d^3x/ds^3=-\frac{\sin\theta\tan\theta}{s^2\sin\beta}\left(\cos\varphi-\frac{\tan\theta\sin\varphi}{\sin\beta}\right)$, $d^3y/ds^3=-\frac{\sin\theta\tan\theta}{s^2\sin\beta}\left(\sin\varphi+\frac{\tan\theta\cos\varphi}{\sin\beta}\right)$

$$1/\rho^2=(d^2x/ds^2)^2+(d^2y/ds^2)^2+(d^2z/ds^2)^2=\frac{\sin^2\theta}{s^2}\left(1+\frac{\tan^2\theta}{\sin^2\beta}\right).$$

$$\frac{1}{\rho^2\sigma}=\begin{vmatrix}\frac{dx}{ds}, & \frac{dy}{ds}, & \frac{dz}{ds}\\ \frac{d^2x}{ds^2}, & \frac{d^2y}{ds^2}, & \frac{d^2z}{ds^2}\\ \frac{d^3x}{ds^3}, & \frac{d^3y}{ds^3}, & \frac{d^3z}{ds^3}\end{vmatrix}=\frac{dz}{ds}\left(\frac{d^2x}{ds^2}.\frac{d^3y}{ds^3}-\frac{d^3x}{ds^3}.\frac{d^2y}{ds^2}\right)$$

$$=\frac{\sin^2\theta\tan\theta}{s^3\sin\beta}\left(1+\frac{\tan^2\theta}{\sin^2\beta}\right)\cos\beta\cos\theta.$$

$$\therefore\ 1/\sigma=\frac{\tan\theta\cos\beta\cos\theta}{s\sin\beta}=\frac{\sin\theta\cos\beta}{s\sin\beta}$$

Also solved by *WILLIAM HOOVER*.

102. Proposed by CHARLES C. CROSS, Whaleyville, Va.

Required the least multiple of 17 which when divided by 2, 3, 4, . . . 16 leaves in each case 1 as a remainder.

I. Solution by MARVIN E. SMITHEY, A. M., Instructor in Mathematics, Randolph-Macon Academy, Bedford City, Va.

$x(2^4.3^2.5.7.11.13)+1$, x being any positive integer, represents all numbers satisfying the given condition. In order to determine what value of x will make this number the least multiple of 17, the following equation must be solved in least positive integers.